A SHORTENED FORM OF SYNTHETIC DIVISION AND SOME OF ITS APPLICATIONS.

By Eugene R. Smith.

The ordinary form of synthetic division is probably familiar to most if not to all teachers of mathematics. I think the best arrangement for division by a binomial divisor is as follows: To divide $2x^3 - 5x^2 + 3x - 7$ by $x - 2$,

$$\begin{array}{r|l}
 & 2 - 5 + 3 - 7 \\
2 & \quad\; 4 - 2 + 2 \\
\hline
1 & 2 - 1 + 1 \,|\, \underline{-5}
\end{array}$$

Giving a quotient $2x^2 - x + 1$ and a remainder -5.

This method of division can equally well be applied to any function of one or two variables, as, $6x^4 - 7x^3y + 5x^2y^2 + 7xy^3 - 11y^4$ divided by $3x^2 - 2xy + 3y^2$

$$\begin{array}{r|l}
 & 6 - 7 + 5 + 7 - 11 \\
-3 & \qquad -6 + 3 + 3 \\
2 & \quad\; 4 - 2 - 2 \\
\hline
3 & 2 - 1 - 1 \;\; |\, \underline{8 - 8}
\end{array}$$

Where the quotient is $2x^2 - xy - y^2$ and the remainder is $8xy^3 - 8y^4$

I might add that I believe in teaching this method of division early in first year algebra, and encouraging its use throughout the course. It will accomplish the necessary division with a minimum of labor and a maximum of speed, neatness of arrangement, and economy of space. The division by a binomial is of the greatest importance, and that form can be still farther shortened without any great increase in difficulty

$$\begin{array}{r|l}
 & 2 - 5 + 3 - 7 \\
2 & 2 - 1 + 1 \,|\, \underline{-5}
\end{array}$$

the work being as follows: Twice 2 and -5 equals -1, twice -1 and 3 equals $+1$, twice 1 and -7 equals -5.

It is this method and some of its applications of which I wish to speak. One of its best uses is in finding plotting values. I will use a cubic equation, but the principal applies equally for

any degree from a quadratic up, if but one variable is involved.

To plot $x^3 - 4x^2 - 2x + 8 = y$

x		y	
-2	$1 - 6 + 10$	-12	
			→ root
-1	$1 - 5 + 3$	$+5$	
0	$1 - 4 - 2$	$+8$	.......(original equation)
1	$1 - 3 - 5$	$+3$	
			→ root
2	$1 - 2 - 6$	-4	
3	$1 - 1 - 5$	-7	
4	$1 \quad 0 - 2$	0	root; depressed equation $x^2 - 2 = 0$

Note that the x and y columns give the plotting values, that a change in sign in the y column shows between what two integers (in the x column) the root lies, and that a zero in the y column shows a root (in the x column), the part between the two columns being the depressed equation, from which in this case the exact values of the other roots can be found. It is evident that this arrangement can with equal value be used in drawing the graph of any function of one variable, or in advanced algebra in solving (for either commensurable or incommensurable roots) an equation of higher degree than the second. Even if Sturm's Theorem were to be used later, this would be the best way to start, and if the location is to be done without Sturm's, there can be no question of its convenience. The method works equally well with fractions, so gives a very simple way to substitute a fractional value.

Again, suppose the roots of an equation are to be diminished by some number, as the roots of $x^3 - 4x^2 - 2x + 8 = 0$ by 2. The work could be arranged

$$\begin{array}{c|llll}
 & 1 - 4 & - 2 & + 8 & \\
2 & 1 - 2 & - 6 & \multicolumn{1}{|l}{- 4} & \\
 & 1 \quad 0 & \multicolumn{1}{|l}{- 6} & & \\
\hline
 & 1 + 2 & & &
\end{array}$$

The new equation being $x^3 + 2x^2 - 6x - 4 = 0$

This would, of course, be applied to Horner's method, and the work would be very much simplified. If a pupil is weak in arithmetic, he would have to drop the shortened form after one

or two decimal places, but I have had pupils who preferred to use it throughout by combining figure by figure, instead of term by term. In handling terms having different signs the Austrian method of subtraction would be used.

Still another application comes in Partial Fractions. If it is required to separate

$$\frac{3x^4 - 2x^2 + x - 1}{(x - 1)^5}$$

into partial fractions, dividing by $x - 1$, and keeping the successive remainders,

$$\begin{array}{r|lllll}
 & 3 & 0 & -2 & +1 & -1 \\
1 & 3 & +3 & +1 & +2 & \boxed{+1} \\
 & 3 & +6 & +7 & \boxed{+9} & \\
 & 3 & +9 & \boxed{+16} & & \\
\hline
 & 3 & \boxed{+12} & & &
\end{array}$$

the answer being

$$\frac{3}{x - 1} + \frac{12}{(x - 1)^2} + \frac{16}{(x - 1)^3} + \frac{9}{(x - 1)^4} + \frac{1}{(x - 1)^5}$$

This method is easily proved, and is, I believe, the shortest way yet found for this kind of example. The degree of the numerator has no effect on it.

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CHEERFULNESS.

If cheerfulness knocks at our door we should throw it wide open, for it never comes inopportunely; instead of that we often make scruples about letting it in. Cheerfulness is a direct and immediate gain—the very coin, as it were, of happiness, and not, like all else, merely a cheque upon the bank.—*Schopenhauer.*